

Probability, Set Theory and Game Theory: A Compact Foundation for Strategic Models

Xiao Yao

[affiliation]

Layout demonstration — sample record, not a published paper.

Abstract

How do three apparently separate branches of mathematics meet in one model? This record arranges sample spaces, set operations and strategic equilibria into a reusable order and says what each step carries into the next.

Keywords: probability; set theory; game theory; equilibrium

1 Introduction

Probability, set theory and game theory are usually taught in three rooms. The cost of that arrangement appears the first time a model needs all three at once: an outcome set, a belief over it, and a rule that maps beliefs to choices.

This record puts them in one order and keeps the notation stable from the first line to the last. The order is not the only possible one; it is the one that makes each later step depend on exactly one earlier step.

The text is a specimen. No theorem is proved and no estimate is reported; the placeholders show where a completed paper would place a proof or a fit.

2 Probability as a measure

The first object is a set of outcomes, written once and named. Everything probabilistic in the rest of the text is a function on that set, which is why the set is fixed before any belief is mentioned.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (1)$$

Writing the measure this way makes the additivity assumption visible. A model that quietly violates it elsewhere in the text will contradict an equation the reader can point at.

3 Sets before strategies

A strategy set is a set before it is a choice. Table 1 keeps the three operations of the previous section next to the strategic object they will later describe, so the translation is never more than one line away.

The middle row carries the most weight in practice: common knowledge is an intersection statement, and most confusion about equilibria is confusion about what is inside it.

Operation	Reading	Later role
union	either outcome	support of a mixed strategy
intersection	both at once	common knowledge
complement	not this one	off-path belief

Table 1: Set operations and their later role in a strategic model. Layout demonstration only.

4 Equilibrium as a fixed point

With outcomes, beliefs and strategy sets in place, an equilibrium is a fixed point of one map: the best response composed with the belief it induces. Stated that way, existence and uniqueness become questions about the map rather than about the story.

The specimen stops at the statement. A completed paper would add the continuity argument, the counterexample that shows the argument is tight, and the comparative static that gives the model its empirical content.

5 What a finished paper adds

A finished version would add proofs, a worked example with numbers, and a section that says which of the three foundations the result actually uses. That last section is the one readers cite when they reuse the model.

References

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